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MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

DMA5201 – MATHEMATICAL TECHNIQUES 2

(All sections / Groups)

7 FEBRUARY 2023

9.00 a.m – 11.00 a.m

(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of five pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. **You are required to write proper steps.**

Question 1 (10 marks)

(a) Find the following limits.

[You must show at least one intermediate step where $\lim_{x \rightarrow c}$ is still needed.]

(i) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{2x^2 - 3x - 2}$

(ii) $\lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{x + 20} - 5}$

(iii) $\lim_{x \rightarrow \infty} \frac{2x^2}{(x + 2)^3}$

[6 marks]

(b) Given $f(x) = \begin{cases} x^2 - 2x - 1 & \text{if } x < 3 \\ x + 2 & \text{if } x \geq 3 \end{cases}$

(i) Determine $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$.

[For this part, you must show at least one intermediate step where $\lim_{x \rightarrow 3}$ or

$\lim_{x \rightarrow 3^+}$ is still needed.]

(ii) Does $\lim_{x \rightarrow 3} f(x)$ exist? Give your reason. If it exists, state its value.

(iii) Is the function $f(x)$ continuous at $x = 3$? Give your reason for your answer.

[4 marks]

Continued.....

Question 2 (10 marks)

- (a) Find $\frac{dy}{dx}$ with y as given.

[Use the product rule or the quotient rule for differentiation; show proper steps.]

(i) $y = \sqrt{x} \sin x$

(ii) $y = \frac{e^{2x}}{\tan x}$

[3 marks]

- (b) A spherical balloon is being inflated at the rate of $10\text{m}^3/\text{min}$. Find the rate at which the radius is increasing when radius is 5m.

[Volume of sphere, $V = \frac{4}{3}\pi r^3$]

[2marks]

- (c) Use **implicit differentiation** to obtain $\frac{dy}{dx}$ for the equation $2y^2 + 3xy - x^3 = 0$

[3.5 marks]

- (d) Given the function $f(x) = 2x^3 - 3x^2 - 36x$, determine the critical numbers.

[1.5 marks]

Continued.....

Question 3 (10 marks)

(a) Integrate the following integral.

$$\int (2x^3 + 2 \sin x - 2\pi) dx$$

[1.5 marks]

(b) Solve the following integral using **integration by substitution**.

$$\int (x^2 + 2x + 4)^3 (x + 1) dx$$

[2.5 marks]

(c) Use integration by partial fraction to find $\int \frac{5x - 3}{x^2 - 2x - 3} dx$.

[4 marks]

(d) Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$.

[2 marks]

Continued.....

Question 4 (10 marks)

- (a) Solve the first order **separable equation** $\frac{dy}{dx} = \frac{x^2 + 1}{2y}$ subject to the initial condition $y(3) = 3$.

[2.5 marks]

- (b) Solve the differential equation, $\frac{dy}{dx} + y = e^x$ subject to the initial condition $y(0) = 1$ using the method of integrating factor.

[3 marks]

- (c) Given the non-homogeneous differential equation $y'' - 3y' + 2y = 4 + x$

(i) Determine the complementary solution, y_c .

(ii) If $y = Ax + B$ is a **particular solution** of the differential equation $y'' - 3y' + 2y = 4 + x$. Determine the values of A and B .

(iii) Hence, write down the **general solution** for the differential equation $y'' - 3y' + 2y = 4 + x$.

[4.5 marks]

Continued.....

Question 5 (10 marks)

(a) Given that the 4th term of an arithmetic sequence is 52 and that the 9th term is 22.

(i) Find the first term and common difference.

(ii) Find the 40th term.

[4 marks]

(b) Compute the sum for the following geometric sequence.

(i) $(2) + (2)^2 + (2)^3 + \dots + (2)^6$

(ii) $\sum_{i=1}^9 3\left(-\frac{1}{4}\right)^i$. Write your answer as a fraction.

[3 marks]

(c) Find the 5th term and 7th term in the expansion of $(3x^2 - y)^{13}$

[3 marks]

End of Page